Frequency Response Analysis (Part – II)

1. In the Bode – plot of a unity feedback control system, the value of phase of $(j\omega)$ at the gain cross over frequency is – 125^0. The phase margin of the system is

(a) – 125^0 (b) – 55^0 (c) 55^0 (d) 125^0 **[GATE 1998: 1 Mark]**

Soln. The Phase angle of $(j\omega)$ at $= \omega_{gc} = -125^0$

ω_{gc} is gain cross over frequency,

$\phi_{gc} = \angle(j\omega_{gc}) = -125^0$

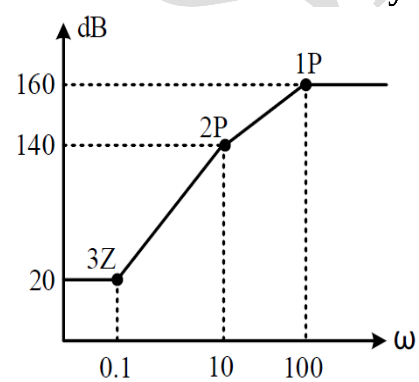
The phase margin $\gamma = 180 + \phi_{gc}$

$= 180 - 125$

$= 55^0$

Option (c)

2. The approximate Bode magnitude plot of a minimum – phase system is shown in the figure. The transfer function of the system is



$(a)\, 10^8 \dfrac{(s+0.1)^3}{(s+10)^2(s+100)}$ $(b)\, 10^7 \dfrac{(s+0.1)^3}{(s+10)(s+100)}$

$(c)\, 10^8 \dfrac{(s+0.1)^2}{(s+10)^2(s+100)}$ $(d)\, 10^9 \dfrac{(s+0.1)^3}{(s+10)(s+100)^2}$

[GATE 2003: 2 Marks]

Soln. Gain changes by (140 – 20) dB when ω changes from 0.1 to 1 and 1 to 10 i.e. 2 decades slope is 60dB/decade

ω = 0.1 change in slope = +60dB/decade 3 real zero

ω = 10 change in slope is from +60dB/decade to 20dB/decade i.e. –40dB/decade 2 real poles

ω = 100 change is slope is from 20dB/decade to 0 dB i.e. – 20dB/decade 1 finite pole

$$T.F = \frac{(1+\frac{s}{0.1})^3}{(1+\frac{s}{10})^2(1+\frac{s}{100})}$$

Magnitude is 20dB at $\omega = 0.1$

$$20\log K = 20|_{\omega=0.1}$$

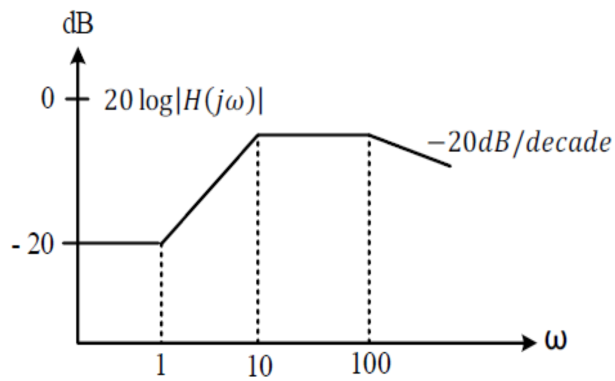
K = 10

$$T.F = \frac{10(1+\frac{s}{0.1})^3}{(1+\frac{s}{10})^2(1+\frac{s}{100})}$$

$$= 10^8 \frac{(s+0.1)^3}{(s+10)^2(s+100)}$$

Option (c)

3. Consider the Bode magnitude plot shown in the figure. The transfer function H(s) is



(a) $\frac{(s+10)}{(s+1)(s+100)}$

(b) $10^2 \frac{(s+1)}{(s+10)(s+100)}$

(c) $\frac{10(s+0.1)}{(s+10)(s+100)}$

(d) $10^3 \frac{(s+100)}{(s+1)(s+100)}$

Soln. At ω = 1 change in slope 20 dB/decade Zero at ω = 1

ω = 10, change in slope 20dB to 0dB i.e. – 20dB/decade pole at ω = 10

ω = 100 change in slope – 20dB/decade pole at ω = 100

Transfer function

$$H(S) = \frac{K(s+1)}{(\frac{s}{10}+1)(\frac{s}{100}+1)}$$

$20 \log K = -20$

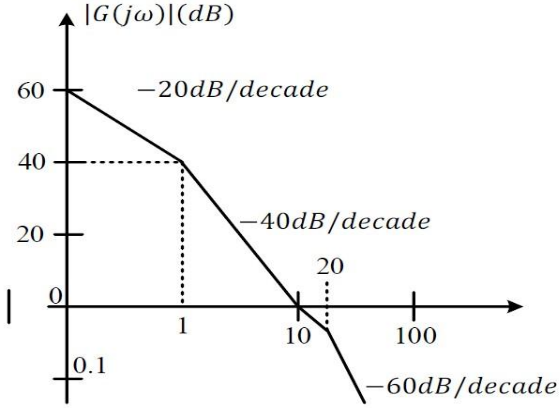
$K = (10)^{-1} = 0.1$

$$H(S) = \frac{0.1(s+1)}{(\frac{s}{10}+1)(\frac{s}{100}+1)}$$

$$\boldsymbol{H(S) = \frac{10^2(s+1)}{(s+10)(s+100)}}$$

Option (b)

4. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function G(s) corresponding to this Bode plot is



(a) $\frac{1}{(s+1)(s+20)}$ (b) $\frac{1}{(s+1)(s+20)}$

(c) $\frac{100}{(s+1)(s+20)}$ (d) $\frac{100}{(s+1)(0.005s+1)}$

[GATE 2007: 2 Marks]

Soln. Pole at s = 0, s = 1, s = 20

$$T.F = \frac{K}{(1+\frac{s}{1})(1+\frac{s}{20})}$$

$60 = 20 \log K\omega|_{\omega=0.1}$

$60 = 20log_{10}K - 20log_{10}\omega, \omega = 0.1$

$60 = 20log_{10}K + 20$

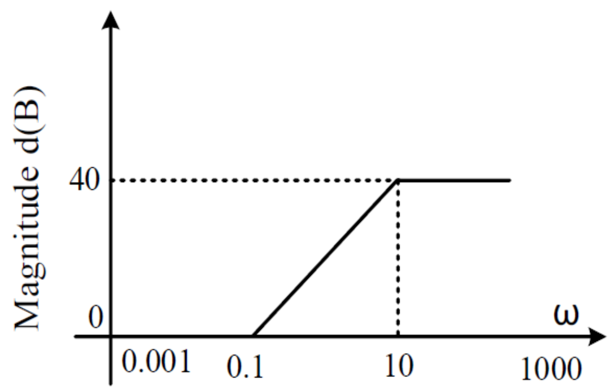
$40 = 20\ log_{10}K$

K = 100

$$T.F \frac{100}{s(s+1)(0.005s+1)}$$

Option (d)

5. For the asymptotic Bode magnitude plot shown below, the system transfer function can be



(a) $\frac{10s+1}{0.1s+1}$ (b) $\frac{100s+1}{0.1s+1}$ (c) $\frac{100s}{10s+1}$ (d) $\frac{10s}{0.1s+1}$ **[GATE 2010: 1 Mark]**

Soln. Slope of asymptote changes from 0 to 40 dB in 2 decades (0.1 to 1, 1 to 10) or slope is 20dB/decade

Zero at ω = 0.1

ω = 10, slope changes from 20dB to 0dB i.e. (-20dB/decade)

Pole at ω = 10

$$\text{T.F} = \frac{K(1+\frac{s}{0.1})}{(1+\frac{s}{10})}$$

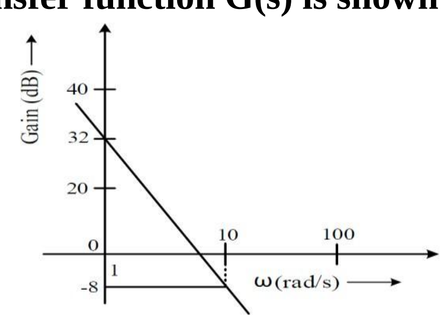
$0dB|_{\omega=0.001} = 20log_{10}K$

K = 1

$$\text{T.F} = \frac{10s+1}{0.1s+1}$$

Option (a)

6. The Bode plot of a transfer function G(s) is shown in the figure below:



(a) $\frac{39.8}{S}$ **(b)** $\frac{39.8}{S^2}$ **(c)** $\frac{32}{S}$ **(d)** $\frac{32}{S^2}$ **[GATE 2013: 1 Mark]**

Soln. Slope = -40dB/decade

2 poles at $\omega = 0$

$20 \log K - 40 log_{10} 10 = -8$

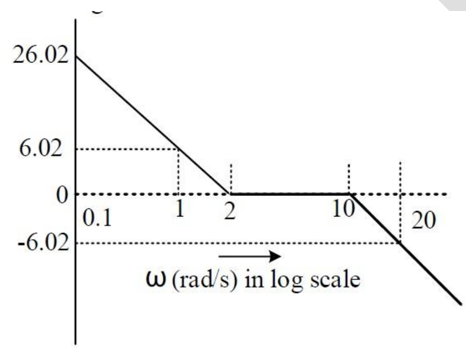
$20 log_{10} K = 32$

$K = 10_{3220}/= 39.8$

$T.F = \frac{39.8}{S^2}$

Option (d)

7. The Bode asymptotic magnitude plot of a minimum phase system is shown in this figure.



If the system is connected in a unity negative feedback configuration, the steady state error of the closed loop system, to a unit ramp input, is_____.

[GATE 2014: 2 Marks]

Soln. Pole at s = 0, s = 10

Zero at s = 2

The open loop transfer function of the system

$$(s)H(s) = \frac{(\frac{S}{2}+1)}{(\frac{S}{10}+1)}$$

$$26.02|_{\omega=0.1} = 20log_{10}K - 20log_{10} \omega$$

$$26.02 = 20log_{10}K - 20log_{10}10^{-1}$$

$$= 20log_{10}K + 20$$

$$6.02 = 20log_{10}K$$

$$K = 1.99 \cong 2$$

$$(s)H(s) = \frac{2*10(S+2)}{2(S+10)}$$

$$G(s)H(s) = \frac{10(S+2)}{S(S+10)}$$

Steady state error for ramp input is $e_{ss} = \frac{1}{K_a}$

$where\ K_a = \lim_{s\to} S(S)H(S)$

$$\boldsymbol{e_{ss} = \frac{1}{2} = 0.5}$$

8. In a Bode magnitude plot, which one of the following slopes would be exhibited at high frequency by 4th order all-pole system?

(a) – 80 dB/decade (b) – 40 dB/decade
(c) + 40 dB/decade (d) + 80 dB/decade **[GATE: 2014 1 Mark]**

Soln. 4th order all-pole system means that the system must be having no zero or s-term in the numerator and S^4 term in denominator.

$$(s) \propto \frac{1}{S^4}$$

One pole exhibits slope of (-20dB/decade), so four pole exhibits slope of -80dB/decade

Option (a)